Topic: Introduction to continuity

Question: Find the points of discontinuity of the function.

$$f(x) = \frac{x - 5}{x^2 - 25}$$

Answer choices:

A $x = -5$

B $x = 5$

C $x = \pm 5$

D $x = 25$

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Solution: C

The denominator of this function can be factored into $(x + 5)(x - 5)$. Therefore, this function has discontinuities at $x = -5$ and at $x = 5$ because the function is undefined at these points (the denominator is equal to 0).

The discontinuity at $x = 5$ is removable because the 0 in the denominator can be divided out of the numerator and denominator; however, the discontinuity at $x = -5$ is non-removable because it cannot be divided out.

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Topic: Introduction to continuity

Question: Find the points of discontinuity of the function.

$$f(x) = \begin{cases} x + 2 & x < -1 \\ \sqrt{x+1} & x \geq -1 \end{cases}$$

Answer choices:

A Discontinuities at all points on the real number line

B $x = -1$

C $x = 1$

D $x = \pm 1$

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Solution: B

This is a piecewise-defined function, which means that, when we test it for continuity, we have to test the continuity of each piece in its own domain, and then test the "break" point where the transition occurs between the pieces.

In our case, the "break" occurs at $x = -1$, because that's where the first piece stops defining the function and the second piece takes over. We'll test the first piece, then the second piece, then the "break" point.

The first piece, $x + 2$, has no discontinuities for $x < -1$ (the domain specified for that piece), because it's a polynomial function, and all polynomials are continuous for all real numbers.

The second piece, $\sqrt{x+1}$, also has no discontinuities for its domain, $x \geq -1$, because this domain ensures that we get 0 or a positive value under the square root sign. If the domain of this piece included any values $x < -1$, we would have to include those as discontinuities, because they'd give us a negative value underneath the square root.

Looking at the "break" point, we can see that we do have a discontinuity at $x = -1$, because the limit does not exist there. The left-hand limit is

$$\lim_{x \to -1^-} f(x) = -1 + 2 = 1$$

and the right-hand limit is

$$\lim_{x \to -1^+} f(x) = \sqrt{-1+1} = 0$$

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Since the left- and right-hand limits are not equal, there is no general limit and we have a discontinuity at $x = -1$.

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Topic: Introduction to continuity

Question: Find the points of discontinuity of the function and state whether each is removable or non-removable.

$$f(x) = \frac{x - 1}{x^2 + x - 2}$$

Answer choices:

A Removable discontinuity at $x = 1$

B Non-removable discontinuity at $x = 1$

C Removable discontinuity at $x = -2$

D Non-removable discontinuity at $x = -2$

E Removable discontinuity at $x = 1$ and non-removable discontinuity at $x = -2$

F Non-removable discontinuity at $x = 1$ and removable discontinuity at $x = -2$

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Solution: E

The denominator factors into $(x-1)(x+2)$, and so the function has discontinuities at $x=1$ and $x=-2$, as these make the function undefined with a 0 in the denominator. The discontinuity at $x=1$ is removable because it divides out with the numerator. However, the discontinuity at $x=-2$ is non-removable because it doesn't divide out.

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